4.6 Summary of Curve Sketching - Day 2

Methods to analyze the graph of a function include:

x-intercepts	domain and range
y-intercepts	symmetry
continuity	differentiability
vertical asymptotes	horizontal asymptotes
relative extrema	increasing/decreasing
points of inflection	concavity
infinite limits at infinity	

Procedure:

1. Find first and second derivatives.
2. Identify the x and y-intercepts, domain, symmetry, horizontal and vertical asymptotes.
3. Identify the critical numbers and possible points of inflection.
4. Set up a chart to determine function characteristics. Determine test intervals around the critical numbers and possible points of inflection. Then find values or signs for each point or interval in the function, first and second derivative. Use this information to determine function characteristics.
5. Sketch the graph of the function.

Example: Analyze and sketch the graph of the function.

$$f(x) = \frac{x^2+1}{x^2-2}$$

$$f'(x) = \frac{(x^2-2)(2x)-(x^2+1)(2x)}{(x^2-2)^2}$$

$$f'(x) = \frac{2x^3-4x-2x^3-2x}{(x^2-2)^2}$$

$$\boxed{f'(x) = \frac{-6x}{(x^2-2)^2}}$$

$$f''(x) = \frac{(x^2-2)^2(-6)+6x[2(x^2-2)(2x)]}{(x^2-2)^4}$$

$$f''(x) = \frac{(x^2-2)[-6(x^2-2)+24x^2]}{(x^2-2)^4}$$

$$f''(x) = \frac{-6x^2+12+24x^2}{(x^2-2)^3}$$

$$\boxed{f''(x) = \frac{18x^2+12}{(x^2-2)^3}}$$

$f(x) = \frac{x^2+1}{x^2-2}$

$f'(x) = \frac{-6x}{(x^2-2)^2}$

$f''(x) = \frac{6(3x^2+2)}{(x^2-2)^3}$

x-intercept: none $\quad 0 = \frac{x^2+1}{x^2-2}$

y-intercept: $(0, -\frac{1}{2})$ $\quad f(x) = \frac{0^2+1}{0^2-2}$

domain: $(-\infty, -\sqrt{2}) \cup (-\sqrt{2}, \sqrt{2}) \cup (\sqrt{2}, \infty)$

Vertical asymptote: $x = \pm\sqrt{2}$

horizontal asymptotes: $y = 1$ $\quad \lim_{x\to\infty} \frac{x^2+1}{x^2-2}$

Symmetry: symmetric to y-axis

$f(x) = f(-x)$

$f(x) = \frac{x^2+1}{x^2-2}$

$f'(x) = \frac{-6x}{(x^2-2)^2}$

$f''(x) = \frac{6(3x^2+2)}{(x^2-2)^3}$

critical #'s:

$x=0, x=\pm\sqrt{2}$

$f'(x) = 0$ or undefined

$f'(x) = \frac{-6x}{(x^2-2)^2}$

$=0 \to x = 0$

und $\to x = \pm\sqrt{2}$

possible points of inflection

$x = \pm\sqrt{2}$

$f''(x) = 0$ or undefined

$f''(x) = \frac{6(3x^2+2)}{(x^2-2)^3} = 0$

$=0 \to$ D.N.E

und $\to x = \pm\sqrt{2}$

$f(x) = \frac{x^2+1}{x^2-2}$

$f'(x) = \frac{-6x}{(x^2-2)^2}$

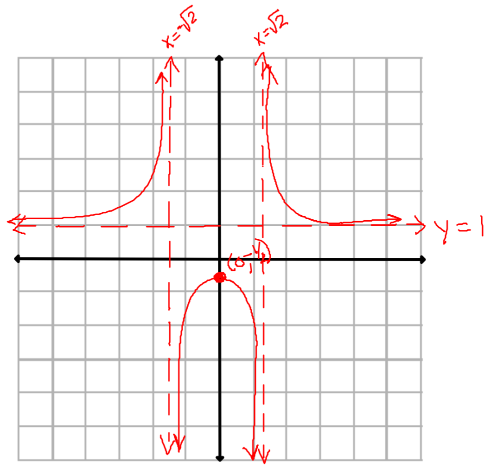
$f''(x) = \frac{6(3x^2+2)}{(x^2-2)^3}$

critical numbers: $x = 0$, $x = \pm\sqrt{2}$

possible points of inflection: $x = \pm\sqrt{2}$

	$f(x)$	$f'(x)$	$f''(x)$	Characteristics
$(-\infty, -\sqrt{2})$	X	+	+	increasing, concave up
$x = -\sqrt{2}$	und	und	und	vertical asymptote
$(-\sqrt{2}, 0)$	X	+	−	increasing, concave down
$x = 0$	$-\frac{1}{2}$	0	−	relative maximum
$(0, \sqrt{2})$	X	−	−	decreasing, concave down
$x = \sqrt{2}$	und	und	und	vertical asymptote
$(\sqrt{2}, \infty)$	X	−	+	decreasing, concave up

$$f(x) = \frac{x^2 + 1}{x^2 - 2}$$



HOMEWORK

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